

Students' lateral mathematical thinking ability on trigonometric problems

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Abstract. To describe students' lateral mathematical thinking ability on trigonometric problems, we conducted a research on 30 students (15/16 year old) in West Java, Indonesia. This research is a qualitative descriptive research and data collection technique is written test with 3 description questions. The test results were analyzed using three indicators of lateral mathematical thinking ability, (1) finding different ways of looking at things, (2) identifying the dominant ideas of the problems, and (3) associating concepts / ideas into several strategies may be true or false. Results of the study showed that: (1) almost one-third of the number of students can look for different ways of looking at a problem, (2) more than one-third of students can identify the dominant ideas of the problem, and (3) more than half students can associate a concept / idea so that it becomes some strategies that may be true or false. Based on the result of research, it is seen that most students do not have good lateral thinking ability in solving trigonometric problems.

1. Introduction

Thinking activities cannot be separated from Mathematics. Mathematics is a science that emphasizes the formation of thinking ability [1]. Mathematics is the discipline of thinking and processing logic, both quantitatively and qualitatively [2]. And the basic objects of mathematics which are facts, concepts, relations/operations, and principles are abstract things so to understand it is not enough just by memorizing but it needs a thought process [3]. Therefore, it can be said that Mathematics is a science that is in dire need of thought activity.

There are two types of thinking: (1) vertical thinking, i.e. the traditional-historical method using logical processes, and (2) lateral thinking, i.e. relatively new types of thinking that complement analytical and critical thinking [4]. De Bono stated that traditional solving methods are analyzing problems, identifying causes, and then proceeding to remove the cause. However, this does not apply to all cases, for example, if there is more than one cause of the problem, or where the cause cannot be found or cannot be deleted. If analysis and argument are not enough, it is necessary to develop a constructive habit of thinking. Lateral thinking is not a substitute for vertical thinking because both are needed and complementary. However, the reality that occurs in education generally tends to use vertical thinking.

In solving a problem, students only imitate what the teacher does and assume that the problem is done just like the example [5]. This is what makes students less able to solve problems with other alternatives and less opportunity to freely express themselves. The 2014 national examination report issued by the Center for Educational Assessment showed that the achievement of trigonometric competency decreased by 27.55% from 2012 to 2013 (from 78.63% to 51.08%) and in 2014 it

increased to 58.41% [6]. From the data shows that students have difficulty in solving trigonometric problems.

Lateral thinking is not something that happens by chance and the technique needs to be taught. The best way to acquire lateral thinking skills is to acquire skills in the use of a set of tools that are all used to produce the same [4]. Openness to the use of provocative and challenging information that is well established and adequate, is the rationale of lateral thinking [4]. Therefore, it allows humans to reconstruct existing patterns.

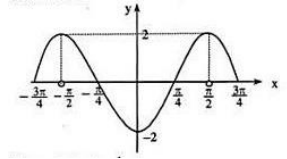
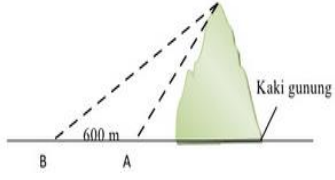
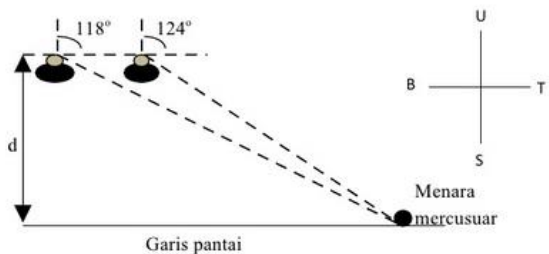
The importance of lateral mathematical thinking ability of students is seen from the results of research conducted by Leonard, i.e. there is the influence of lateral thinking ability on student achievement [7]. Gencel also mentioned that students' lateral thinking tendency is at medium level [8]. In addition, there are various kinds of students' lateral thinking processes in solving open-ended problems [9].

Based on students' difficulties in solving trigonometric problems and the importance of lateral mathematical thinking skills in education, this study aims to describe students' lateral mathematical thinking in solving trigonometric problems.

2. Method

This research is a qualitative research with descriptive problem because the purpose of this research is to describe or explore a symptom, event, incident thoroughly and deeply [10]. The data was collected by using the test instrument that was tested on 30 students (15/16 year old) in West Java, Indonesia. The test results were analyzed using three indicators of lateral mathematical thinking ability, (1) finding different ways of looking at things, (2) identifying the dominant ideas of the problems, and (3) associating concepts / ideas into several strategies may be true or false. The written test is given in Table 1.

Table 1. The instrument of lateral mathematical thinking ability

No	Mathematics problems
1	<p>Indicator: Finding different ways of looking at things Make a question using the picture beside then answer it completely (you can give additional information to your question).</p> 
2	<p>Indicator: Identifying the dominant ideas of the problems To measure the height of a mountain, an observer uses the scheme as in the picture on the side. First the observer is at A and observes the mountain at an elevation angle of 600, then walks away from the mountain and stops at B 600 m from A. At B he observes a mountain with an elevation angle of 300. The mountain's foot is 1,800 m above sea level. Is it enough data to calculate the height of the mountain above sea level? If it's enough, finish it! If not, what data do you think is lacking? Complete and finish it.</p> 
3	<p>Indicator: Associating concepts / ideas into several strategies may be true or false A boat is sailing eastward (parallel to the shoreline) with a speed of 21 km/h. At a certain moment, the direction of the boat to the lighthouse tower is 118° and 20 minutes later its direction is 124°. Determine the distance of the boat, d, from the shoreline, if the lighthouse tower is located on the shoreline!</p> 

3. Result and Discussion

The result of the students' lateral mathematical thinking test on trigonometric material can be seen in Table 2.

Table 2. Indicators and student achievement

Indicators of lateral thinking ability	Percentage of correct answers
Finding different ways of looking at things	27,33%
Identifying the dominant ideas of the problems	37%
Associating concepts / ideas into several strategies may be true or false	61,54%

Based on Table 2, the average percentage of student work result is 41.95%.

3.1 Finding different ways of looking at things

In this indicator, students are expected to be able to solve problems with various strategies of completion by using logic, reasoning, or other strategies. This indicator is contained in Problem 1 in Table 1. Most students make inquiries according to the information contained in the drawings and the questions are not wrong, but the question is not complex / deep. The best answer from the students is shown in Figure 1, while the other students' answers are shown in Figure 2 and Figure 3.

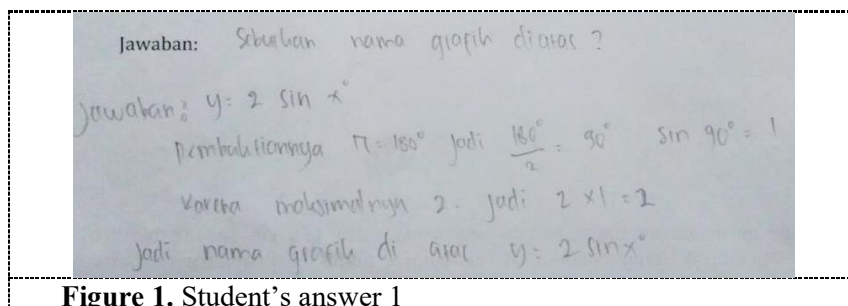


Figure 1. Student's answer 1

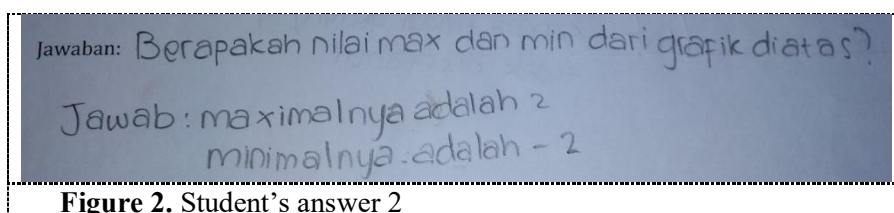


Figure 2. Student's answer 2

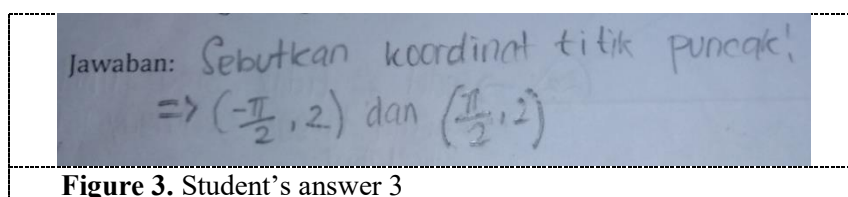


Figure 3. Student's answer 3

From the student's answer in Figure 1, it appears that the answer to the question made is not correct. As in Figures 2 and 3, it appears that students make inquiries but use only partial information and do

not use the numbers and images available. This is done by most students. The average percentage of students' lateral mathematical thinking ability on this indicator is still low at 27.33%.

3.2 Identifying the dominant ideas of the problems

In this indicator, students are expected to write down any information obtained from the problem given so that students understand the problem well. This begins with what information is known and what matters are being asked or problematic in the problems presented. This indicator is contained in problem 2 in Table 1.

Handwritten solution for finding the height of a mountain:

$$\frac{600}{\sin 30^\circ} = \frac{BC}{\sin 120^\circ} \Rightarrow \frac{600}{1/2} = \frac{BC}{1/\sqrt{3}} \Rightarrow \frac{1}{2} BC = 600 \cdot \frac{1}{2} \sqrt{3} \Rightarrow BC = 600\sqrt{3} \text{ m}$$

$$\frac{AC}{\sin 30^\circ} = \frac{600}{\sin 30^\circ} \Rightarrow \frac{AC}{1/2} = \frac{600}{1/2} \Rightarrow \frac{1}{2} AC = 600 \cdot \frac{1}{2} \Rightarrow AC = 600 \text{ m}$$

$$\frac{600}{\sin 90^\circ} = \frac{CD}{\sin 60^\circ} \Rightarrow \frac{600}{1} = \frac{CD}{1/\sqrt{3}} \Rightarrow CD = 600 \cdot \frac{1}{2} \sqrt{3} = 300\sqrt{3}$$

Jadi tinggi gunung dari permukaan laut

$$BC + 200\sqrt{3} = 2319.62 \text{ m}$$

Figure 4. Student's answer 1

Data yg diketahui: hanya sudut saja,
jadi saya tidak bisa menghitung
tinggi gunung.

Figure 5. Student's answer 2

Tidak cukup, Karena :
a. Berapa jarak A dan sudut imajiner?
b. Apakah 60° itu sudut dari titik A saja atau gabungan? (karena gambarnya kurang sesuai dengan besar sudutnya)

Figure 6. Student's answer 3

Figure 4 shows that the student is able to solve the problem with the correct answer details. This means that students are able to identify the information that is known and the things that are asked of the problem given. Students can choose what information is used to solve the problem given so they can find the important information that is the key to solving the problem. While based on Figure 5 and Figure 6, students have not been able to identify the dominant idea of the problem at hand. It appears that the student replied that the available data was not sufficient to solve the given problem. From Figure 5, the students mentioned that the known data is only the angle, whereas in the case there are other information such as point spacing A and point B. While in Figure 6, the students mentioned that

the available data is not enough to solve the problem because it does not know the distance between point A and the imaginary angle, and cannot identify whether 60° is the magnitude of angle A only or not. The average percentage of students' lateral mathematical thinking ability on this indicator is still low at 37%.

3.3 Associating concepts/ideas into several strategies may be true or false

In this indicator, students are expected to be able to open up with the concepts used in solving the given problem and using many concepts. This indicator is contained in Problem 3 in Table 1. Solving this problem should use the concept of speed, but none of the students used the concept.

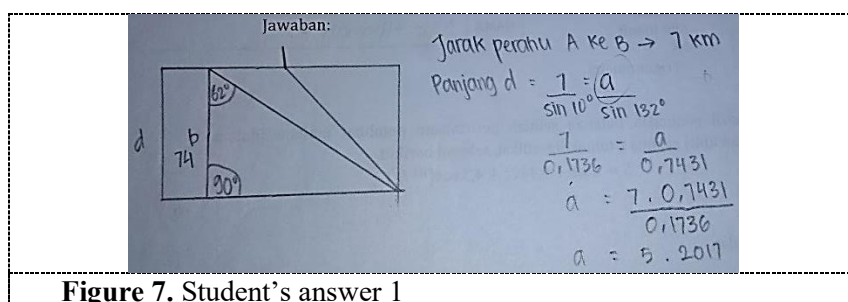


Figure 7. Student's answer 1

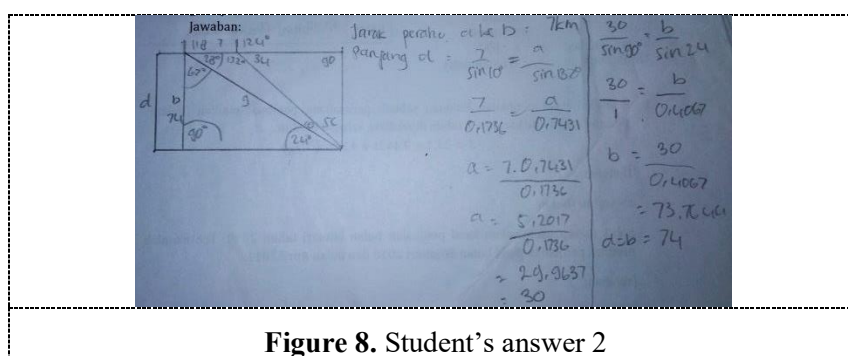


Figure 8. Student's answer 2

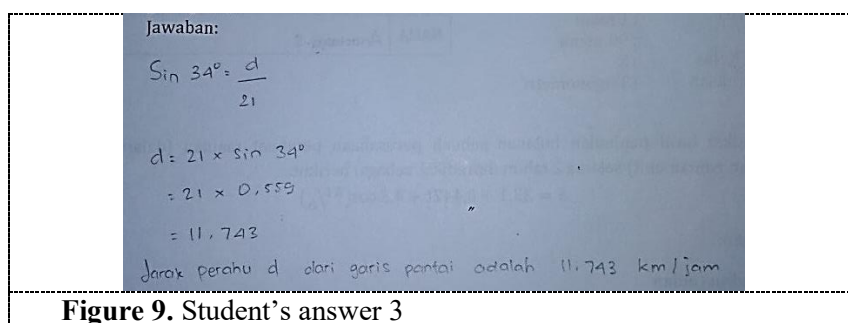


Figure 9. Student's answer 3

Based on Figures 7 and 8, it appears that students writing distance A to B is 7 km. It is true that the distance A to B is 7 km, but the student does not explain how it can happen. Students did not mention point A and point B in the created image. From Figure 7, Figure 8, and Figure 9, it appears that students use the concept of sinus and sinus rules in solving the given problem, but the steps of completion are not appropriate because there is still less data and should be sought. In addition, from Figure 9 it is found that students do not understand that the units of distance are not km/h.

Overall, most students have not been able to find different ways of looking at things, still difficult to identify the dominant ideas of the problems, and have not been able to associating concepts/ideas into several strategies may be true or false.

According to Asmin [7], lateral thinking ability has a role in breaking away from the old conception and making a change of attitude and approach to observing problems in different ways. A person who has lateral thinking ability, then the person has the ability to find new ideas and perceptions and remember that the pattern of the problem is not always symmetrical. It means that using lateral thinking ability, something impossible to use becomes possible in solving problems. Besides, making the best step in problem solving depends on students' lateral thinking ability [7]. This is supported by the results of Leonard's research which states that lateral thinking ability gives influence and benefits to the achievement of student learning achievement.

Based on the result of research, it is seen that most students do not have good lateral thinking ability in solving trigonometric problems. From this research also found that students' thinking ability in solving problems is different, so that different treatment is needed in doing the learning.

4. Conclusion

Based on the results of research and discussion, it can be concluded that the average percentage of students' lateral mathematical thinking ability of the three indicators of lateral thinking is low, i.e. 41.95%. Because of lateral thinking ability is very important for students, teachers need to innovate the learning, both media and learning methods, so that students' lateral thinking ability can develop well.

5. Acknowledgments

This study was funded by Indonesian Endowment Fund of Education (LPDP). We would like to thank the teachers and students for their participation in this research.

6. References

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